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Determination of heat transfer coefficients for hydrogen condensation in condensers of various geometries[1]

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Abstract — An important task for the hydrogen isotopes separation by cryogenic distillation is to establish the shape and dimension of the column condenser and boiler in order to obtain the desired load and separation for the distillation column. In the paper we present the set-up and experimental values for the heat transfer coefficient on various types of condensers. The heat transfer coefficients were determined by measurements on liquid hydrogen flow-rate condensed on the cold surface and temperature drop between the cooling liquid and the condensate. The experiments were made for different vapor pressures and certain temperatures of the cooling liquid from the condenser. As results we determined the condensation heat transfer coefficients for different shapes and geometries of the condensers as a function of the condensate film temperature drop. © 2000 Éditions scientifiques et médicales Elsevier SAS

condensing heat transfer coefficients / plate-fin condenser / hydrogen condensation

Nomenclature

a, b	coefficients in the Redlich–Kwong equation of state	
A^*, B^*, C^*, D^*	coefficients from Mittelhauser equation for saturation pressure	
A	heat transfer area	m^2
C	specific heat	$J\cdot kg^{-1}\cdot K^{-1}$
d	thickness of plate-fin condenser	m
g	acceleration of gravity	$m\cdot s^{-2}$
G	depth of plate-fin condenser	m
h	heat transfer coefficient	$W\cdot m^{-2}\cdot K^{-1}$
H	specific enthalpy	$J\cdot kg^{-1}$
k	thermal conductivity	$W\cdot m^{-1}\cdot K^{-1}$
L	length of surface	m
p	pressure	$N\cdot m^{-2}$
Pr	Prandtl number	
Re	Reynolds number	
Q	heat flow	W
r	radius of tube condenser	m
t	temperature	K
v	specific volume	$m^3\cdot kg^{-1}$

Greek symbols

μ	viscosity	$N\cdot s\cdot m^{-2}$
ρ	density	$kg\cdot m^{-3}$
λ	latent heat	$J\cdot kg^{-1}$

Subscripts

b	boiling
c	condensing
f	film
i	inside
o	outside
O	overall
l	liquid
p	constant pressure
sv	saturated vapor
vap	vaporization
w	outside wall
w′	inside wall

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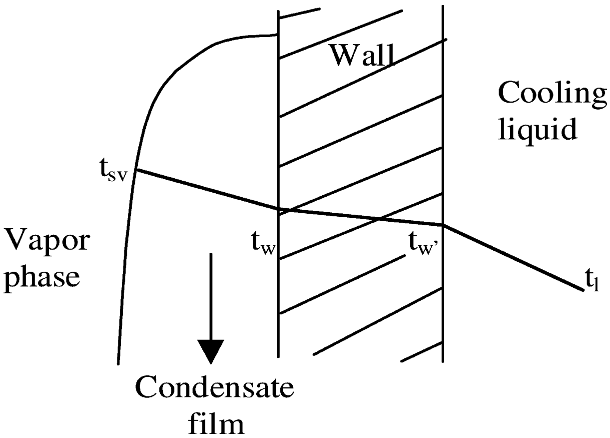

Figure 1. Condensation on a vertical plate

1. INTRODUCTION

The experimental data regarding heat transfer coefficients at hydrogen condensation in condensers cooled with liquid hydrogen [1, 2] show a strong deviation from the values predicted by Nusselt theory. This deviation is especially stronger at small temperature differences between condensing vapors and cooling liquid.

Having in view these data we consider that the deviation of heat transfer coefficients from Nusselt theory is largely determined by the heat resistance of the condensate on the condensing surface. In this way we compared the Nusselt theory predictions with experimental data for tubular condensers with the same inner diameter, the same total condensing area and different heights. In the paper are also presented comparative experimental data for a tubular condenser and a plate-fin type condenser, with the same height and the same total condensing area. The experimental set-up also allowed extending the measurements for heat transfer coefficients using pressurized hydrogen as cooling liquid.

2. BASIC CONSIDERATIONS

The process of condensation on a vertical wall is presented in *figure 1*. Normally the condensate on the wall is the major resistance to the heat transfer on the condensing side [3].

For a smooth surface the condensate usually forms a continuous film on the wall.

Nusselt developed a theoretical relation for the mean coefficient of heat transfer for a pure vapor condensing on a vertical surface:

$$h = 0.943\left[\frac{k^3\rho^2 g\lambda}{L\mu(t_{\mathrm{sv}} - t_{\mathrm{w}})}\right]^{1/4} \tag{1}$$

The basic assumptions in Nusselt's theory are: (1) the surface is isothermal; (2) the flow of the condensate is laminar; (3) vapor and condensate are essentially at the same temperature; (4) only gravitational and liquid forces act on the condensate.

Rohsenow discarded the assumption of negligible convection in the condensate film and included the buoyancy force acting on the condensate film [3]. For the range

$$0 < \frac{C_{\mathrm{p}}(t_{\mathrm{sv}} - t_{\mathrm{w}})}{L} < 1.0$$

the following equation for an average coefficient was proposed:

$$h = 0.943 \cdot \left[\frac{k^3\rho_{\mathrm{l}}(\rho_{\mathrm{l}} - \rho_{\mathrm{sv}})g\lambda(1 + 0.68C_{\mathrm{p}}(t_{\mathrm{sv}} - t_{\mathrm{w}})/\lambda)}{L\mu(t_{\mathrm{sv}} - t_{\mathrm{w}})}\right]^{1/4} \tag{2}$$

Under steady-state conditions with one-directional heat flow, the quantity of heat transferred from the condenser surface and the boiling liquid in condenser is

$$Q_{\mathrm{b}} = h_{\mathrm{b}}A_{\mathrm{b}}\Delta t_{\mathrm{b}} \tag{3}$$

Similarly for a condensing vapor

$$Q_{\mathrm{c}} = h_{\mathrm{c}}A_{\mathrm{c}}\Delta t_{\mathrm{c}} \tag{4}$$

A_{b} and A_{c} are the heat transfer areas and are specific for the type of condenser investigated.

It can be seen from *figure 1* that the thickness of the boiling and the condensing films are not constant along the condenser surface, and therefore the heat transfer coefficients from (3) and (4) represent average values.

The heat transferred is related to the latent heat of condensation and the flow-rate of condensed vapor.

The density of the saturated vapor was computed with the aid of the Redlich–Kwong equation of state:

$$p = \frac{RT}{v - b} - \frac{a}{\sqrt{T}v(v + b)} \tag{5}$$

The vaporization energy was computed starting from Clapeyron–Clausius equation:

$$\frac{\mathrm{d}p_{\mathrm{sv}}}{\mathrm{d}T} = \frac{\Delta H_{\mathrm{vap}}}{T\Delta v_{\mathrm{vap}}} \tag{6}$$

For hydrogen and its isotopes, the pressure on the saturation curve varies with the temperature after the relation proposed by Mittelhauser [4]:

$$\log p_{\mathrm{sv}} = A^* + \frac{B^*}{T} + C^* \log T + D^* \frac{p}{T^2} \qquad (7)$$

For normal hydrogen, p is expressed in mmHg and the coefficients of Mittelhauser equation have the values: $A^* = 4.73642$, $B^* = -47.041$, $C^* = 0.31609$, $D^* = 0.02079$. With (6) and (7), we can finally express

$$\lambda = \Delta v \left[\frac{C^* T^2 - B^* T - 2D^* p}{(T^2/p - D^*)} \right] \qquad (8)$$

where Δv is the difference between the specific volume of the vapor and liquid phase on the saturation curve and was computed using the assumptions presented in [5].

For the other properties involved in the expression of the condensing heat transfer coefficient (1), i.e. thermal conductivity, viscosity, we refer to [6].

3. EXPERIMENTAL SET-UP AND PROCEDURE

The overall view of the experimental set-up is shown in *figure 2*. It consists of a cryogenerator for hydrogen liquefaction (Phillips PPH type) that assures the cooling liquid for hydrogen vapor condensation.

The experimental set-up was designed with the aim to determine the dependence of the condensation heat-transfer coefficient with:

(a) the temperature difference between vapor and cold surface of the condenser when it a certain value of pressure is maintained in the condenser;

(b) pressure of vapor that condenses for different values of temperature difference between vapor and cold surface.

As it can be seen from *figure 2*, the experimental set-up also comprises two electrical heaters. The heater's power was correlated to the condensing power of the cryogenerator and allowed performing experiments in the two ways mentioned above.

In order to reduce the cold losses, the experimental set-up was equipped with a liquid nitrogen shield.

After a high vacuum of approximately $5{\cdot}10^{-5}$ mbar was established and the liquid nitrogen bath was filled, the preliminary steps for the experiments were:

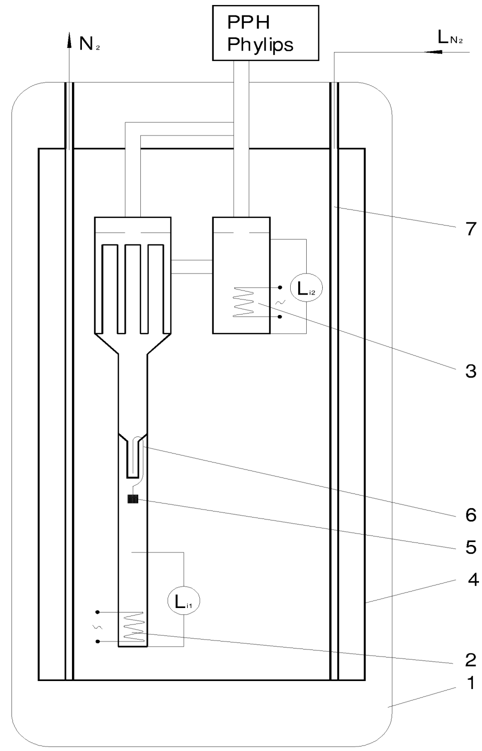

Figure 2. Experimental set-up. Legend: 1. cold-box; 2. heater; 3. heater; 4. radiation shield; 5. cryoresistance; 6. flow-meter; 7. liquid nitrogen bath.

- continuously feeding with purified hydrogen at a constant pressure value (2 bar) allowed getting the necessary level of liquid hydrogen in the column condenser. At this pressure the cryogenerator power is approximately 100 W;
- in the same time keeping 3 bar pressure in the column the desired amount of liquid hydrogen in the column boiler was realized.

The two desired levels of liquid hydrogen in the boiler and condenser were measured by the differential pressure from the two points located on the column and on the condenser. After we obtained the desired levels, heat was gradually introduced with the aid of heater 2 from the bottom of the column until the pressure of the liquid hydrogen in the condenser had a value of 2 bar. The steady state was attained within approximately an hour. After that, the following parameters were measured:

- using simultaneous vapor-pressure measurements of the condensate and boiling liquid the boiling liquid and vapor temperature was measured. The temperature is related with the vapor pressure using equation (7);

• the temperature difference Δt_b between the wall and the temperature of the cooling liquid was measured with a thermocouple with one junction located on the vertical surface—middle point—and the other junction located in the liquid hydrogen;

• the flow-rate of the condensed hydrogen was measured.

For the two condenser types investigated (multi-tube and plate-fin) we considered the temperature drop across the cold wall $(t_w - t_{w'})$ in the following manner:

$$\Delta t_w = -\frac{Q \ln(r_O/r_i)}{2\pi k L} \tag{9}$$

for the tubular condenser and for plate-fin condenser

$$\Delta t_w = -\frac{Qd}{kLG} \tag{10}$$

The temperature drop in the condensate film Δt_c was expressed in the following way:

$$\begin{aligned} \Delta t_c &= \Delta t_o - \Delta t_b - \Delta t_w \\ \Delta t_o &= t_{sv} - t_l \\ \Delta t_b &= t_{w'} - t_l \end{aligned} \tag{11}$$

Δt_w was computed using (9) or (10) where Q is the product of the measured flow-rate and the enthalpy of vaporization expressed by (8). The condensation heat transfer coefficient was computed with (1).

The value of the temperature drop in the condensate film had a maximum for the determination, when we balanced the cryogenerator power only by heater 2.

After that, the amount of heat introduced in the bottom of the column was decreased in small steps. In order to maintain the same value of the pressure for the cooling liquid—2 bar—we introduced small amounts of heat in the heater from the column condenser 3. The refrigeration power of the cryogenerator is balanced by the condensation power plus the heat introduced into the condenser by heater 3. In this way the condensation power of the condenser was decreased, maintaining a constant value of the temperature in the cooling liquid at the same refrigeration power supplied by the PPH cryogenerator.

The amount of vapor condensed on the cold surface of the condenser is measured with the aid of a liquid hydrogen flow meter. This flow meter consists of a collector with a calibrated volume, which has a downcomer. When the collector is filled its complete contents is emptied over a cryoresistance. This is recorded by the variation of the cryoresistance's electrical potential when the liquid hydrogen flows over it. The flow-rate was determined taking into account the volume of the collector and the number of emptyings during a certain time.

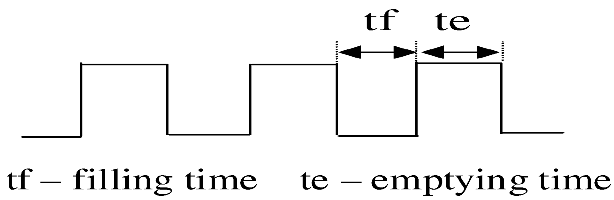

Figure 3. Shape of the recorded voltage pulses from cryoresistance.

Regarding the experimental errors, in the presented experiments, there were two kinds: errors from the flow-rate measurements and errors from the measurement of temperature difference between the wall and the cooling liquid Δt_b.

Theoretically the recorded voltage pulses from the cryoresistance would look in the manner of *figure 3*.

Owing to the deviations from this shape, the calibrated volume was designed in such a way that for a measuring time of 10 minutes, the error is 1 %. In the experiments, the measuring time for each steady state was between 30 minutes and 1 hour. In this way it can be considered that error for flow-rate measurement is lower than 1 %.

In order to estimate the error related to Δt_b measurements, at the beginning of the experiments, after the liquid levels in the condenser and boiler were achieved, the pressure in the cooling liquid and in the condensing vapor were equalized for several times. The corresponding average value of potential to Δt_b was 0.5 μV and we corrected the measured values that were used in equation (11). The accuracy of the millivoltmeter HP-3458A used for the measurements was 50 nV. We can consider that the error for Δt_b measurements varies from 7 % for a temperature difference of 0.18 K between the cooling liquid and vapor down to 3.5 % for a temperature difference of 6.6 K.

4. RESULTS

The experiments were made in the manner presented above for different values of vapor flow-rate for the case (a) of experiments.

We investigated the heat transfer coefficients both for multi-tube condensers (*figure 4*) and for a plate-fin condenser (*figure 5*).

The heat transfer area for a multi-tube condenser is the total inside area of the tubes. As for the plate-fin

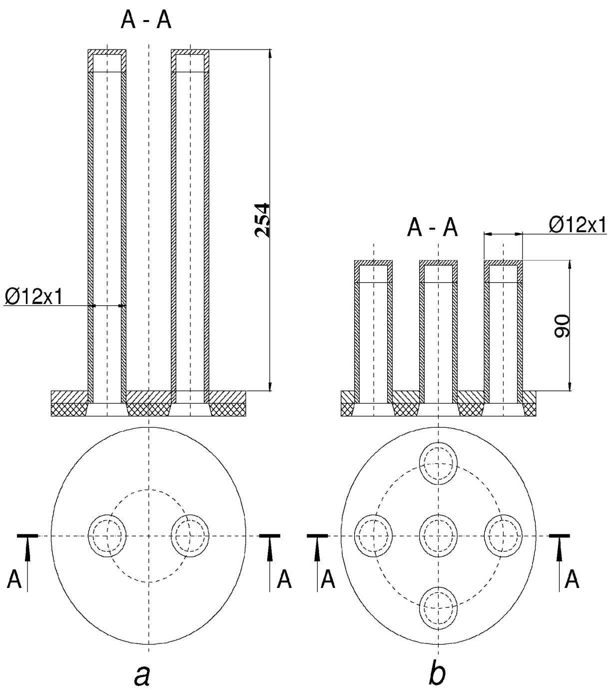

Figure 4. Multi-tube condenser.

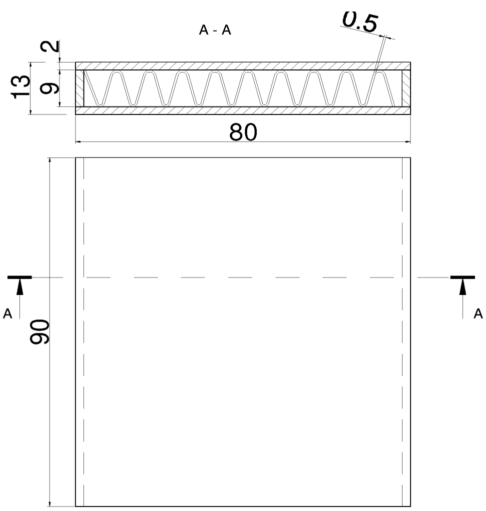

Figure 5. Plate-fin condenser.

condenser, for a singled banked passage arrangement the effective area for heat transfer is given by [7]

$$A_c = A_1 + A_2\eta_2 \tag{12}$$

where subscript 1 denotes the primary surface (parting shot) and subscript 2 denotes the secondary surface (fin).

The fin efficiency of the secondary surface, η_2, is [7]

$$\eta_2 = \frac{\tanh es}{es}, \qquad e^2 = \frac{2h}{k_2 d_2} \tag{13}$$

where h is the local heat transfer coefficient, d_2 is the fin thickness and k_2 is the fin thermal conductivity, s is generally assumed to be half of the fin height.

As for the heat transfer coefficient, it was computed according to

$$h = \frac{k}{2}\left[\frac{Re\,Pr}{L}\right] \tag{14}$$

where Reynolds and Prandtl numbers were computed for the falling liquid film.

The experimental results obtained for the vapor flow-rate and also the condensation heat transfer coefficients computed with equation (4) are presented in *table I*.

Figure 6 shows experimental heat transfer coefficients as a function of temperature drop for the three types of condensers and also the values obtained using the Nusselt criterion (1). As it can be seen from the figure the condensation heat transfer coefficients for all types of condensers investigated were lower than those predicted by Nusselt theory.

Figure 7 shows experimental and calculated heat flux for condensing hydrogen as a function of temperature difference between the cooling liquid and vapor.

Nusselt's equation can be rearranged as a function of film Reynolds number according to [8]

$$\frac{h}{k_f}\left[\frac{\nu_f^2}{g}\right]^{1/3} = 1.47\,Re_f^{-1/3} \tag{15}$$

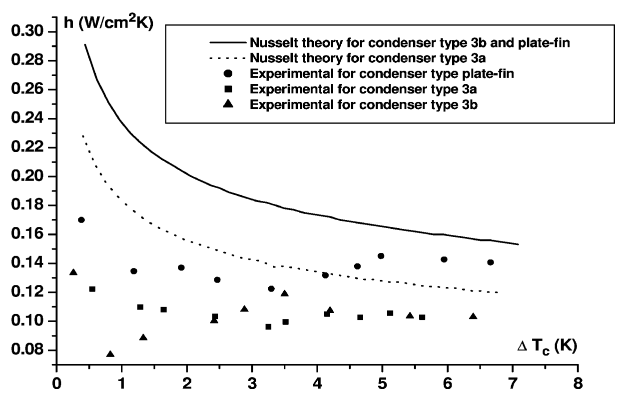

Figure 6. Condensing heat transfer coefficient for the condensers investigated in this work as a function of the condensate film temperature drop.

TABLE I

Condenser type	Vapor pressure (bar)	Vapor temperature (K)	Flow-rate ($g \cdot s^{-1}$)	ΔT_w (K)	ΔT_b (K)	ΔT_c (K)	ΔT_o (K)	Q (W)	h_{exp} ($W \cdot cm^{-2} \cdot K^{-1}$)
Plate-fin	2.1	23.37	0.02	0.25	0.05	0.383	0.18	9	0.16996
	2.4	23.86	0.05	0.5833	0.07	1.187	0.67	21	0.13462
	2.7	24.28	0.08	0.9167	0.1	1.915	1.1	33	0.13699
	3	24.67	0.09	1.0833	0.1	2.467	1.48	39	0.12857
	3.5	25.27	0.12	1.3333	0.12	3.297	2.08	48	0.12244
	3.9	25.74	0.15	1.7222	0.15	4.129	2.56	62	0.13166
	4.1	25.98	0.18	1.9722	0.15	4.618	2.8	71	0.13796
	4.3	26.22	0.2	2.1944	0.25	4.981	3.04	79	0.14511
	4.8	26.83	0.23	2.5	0.2	5.949	3.65	90	0.14272
	5.3	27.45	0.26	2.6944	0.3	6.66	4.27	97	0.14064
Multi-tube type 3b	2.1	23.37	0.01	0.1264	0.05	0.26	0.18	0.98	0.13341
	2.4	23.86	0.02	0.1769	0.05	0.801	0.67	1.372	0.06061
	2.6	24.15	0.04	0.4297	0.06	1.332	0.96	3.332	0.08846
	3.1	24.79	0.08	0.8847	0.07	2.421	1.61	6.86	0.10022
	3.3	25.03	0.11	1.1375	0.1	2.884	1.85	8.82	0.10815
	3.5	25.27	0.14	1.5166	0.1	3.501	2.08	11.76	0.11881
	4	25.86	0.16	1.643	0.12	4.199	2.68	12.74	0.1073
	4.7	26.71	0.21	2.0475	0.15	5.424	3.53	15.876	0.10353
	5.2	27.33	0.25	2.4014	0.15	6.394	4.14	18.62	0.103
Multi-tube type 3a	2.2	23.54	0.02	0.2463	0.05	0.552	0.36	5.39	0.12231
	2.5	24	0.05	0.515	0.05	1.286	0.82	11.27	0.10981
	2.7	24.28	0.07	0.6494	0.1	1.648	1.1	14.21	0.10807
	3.1	24.79	0.1	0.9181	0.09	2.434	1.61	20.09	0.10343
	3.6	25.39	0.12	1.142	0.09	3.254	2.2	24.99	0.09624
	3.7	25.5	0.14	1.2763	0.08	3.516	2.32	27.93	0.09954
	4	25.86	0.17	1.5898	0.11	4.156	2.68	34.79	0.10491
	4.3	26.22	0.19	1.7465	0.12	4.664	3.04	38.22	0.1027
	4.5	26.46	0.22	1.9704	0.13	5.121	3.28	43.12	0.10552
	4.8	26.83	0.24	2.1048	0.14	5.614	3.65	46.06	0.10281

where Re_f is the Reynolds number in the film. The left side of this equation represents the condensation number and having in view that the Nusselt equation is derived for laminar flow, the film Reynolds number at the lowest part of the condensing tube must be less than 1 800.

Figure 8 shows the condensation number as function of Reynolds number for hydrogen in two geometries: plate-fin and multi-tube type 3b.

5. CONCLUSIONS

In *figures 9* and *10* the ratio $(h_{\text{experimental}} - h_{\text{Nusselt}})/h_{\text{Nusselt}}$ as a function of the temperature difference between cooling liquid and vapor is presented. In *figure 9* the experimental data are from the present paper, while in *figure 10* the experimental data are from [1, 2]. Referring to *figure 10* the geometrical characteristics of the tubular condensers investigated were:

- reference [1]—inner diameter: 12.32 mm; height: 112.7 mm; number of tubes: 1;
- reference [2]—inner diameter: 20 mm; height: 200 mm; number of tubes: 8.

It can be seen that the deviations of the heat transfer coefficient values from the Nusselt theory are greater for small temperature differences between the cooling liquid and vapor. Analyzing the dependence from *figure 10*, the heat transfer coefficients for tubular condensers with greater heights (200 mm) are closer to Nusselt theory than the ones for tubular condensers with lower heights, 112.7 mm respectively. The same conclusion arises from the experimental data from the present paper, greater heat

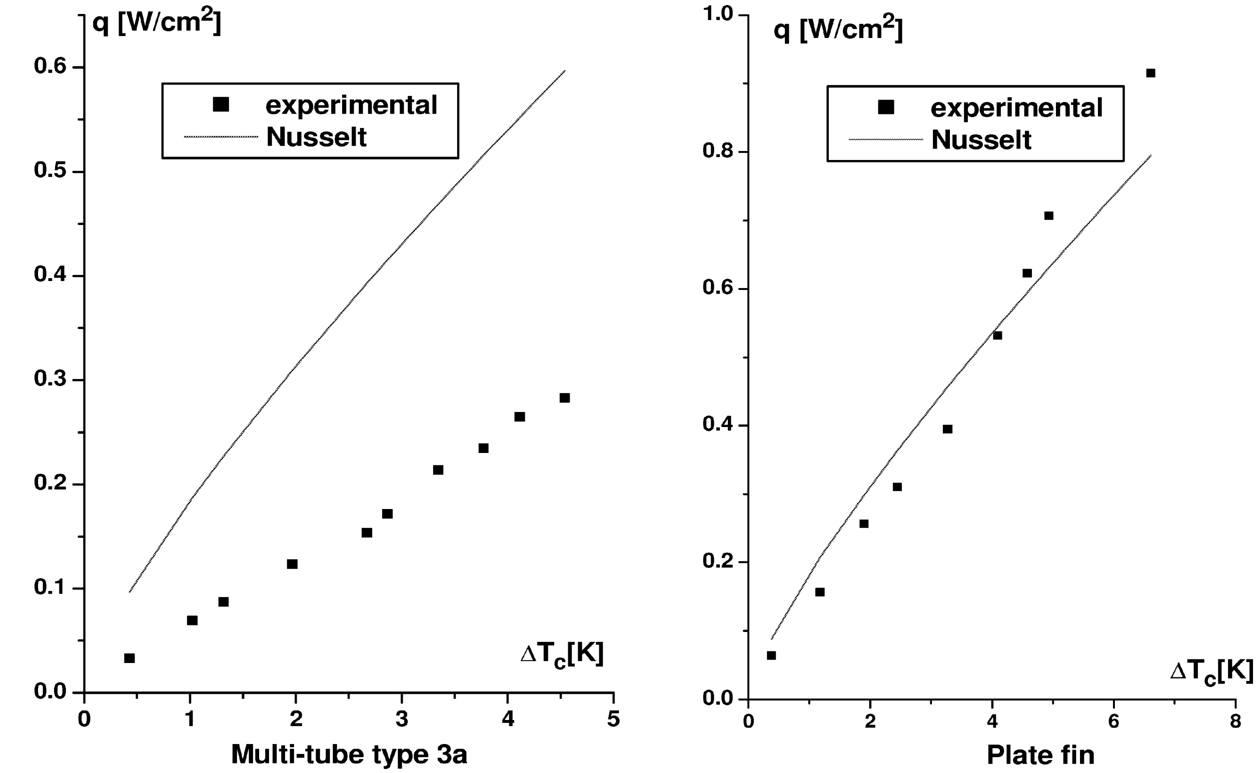

Figure 7. Heat flux as a function of temperature difference across the condensing film for plate-fin condenser.

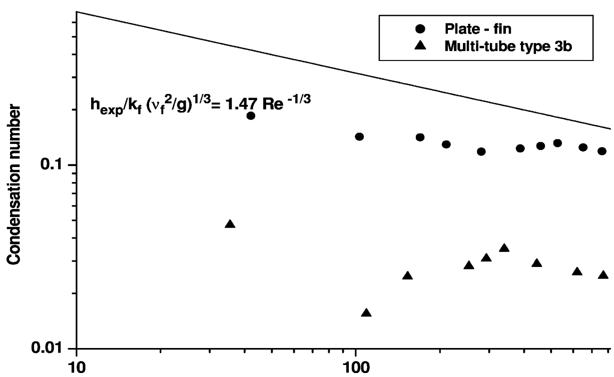

Figure 8. Condensation number as a function of film Reynolds number.

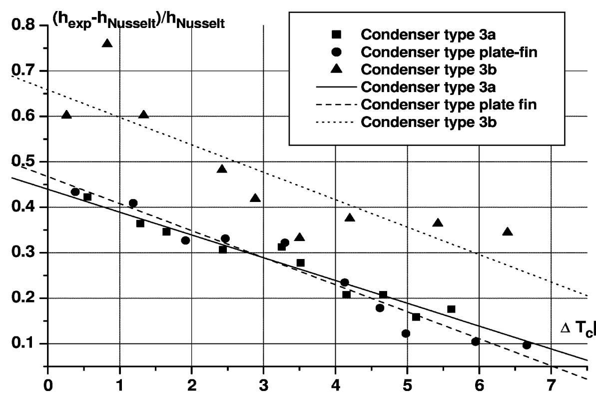

Figure 9. Heat transfer coefficient deviation form Nusselt's theory as a function of the condensate film temperature drop; this work.

transfer coefficients for the tubular condenser of 254 mm, comparing to the tubular condenser of 90 mm.

From the experimental data it can also be concluded that for a condenser of plate-fin type of the same height with a tubular condenser, the heat transfer coefficients are closer to the ones predicted by the Nusselt theory.

It is also important to notice the same behavior of the heat transfer coefficients with the temperature difference between cooling liquid and vapor even if there is one difference in the experimental procedure: in [1, 2] the cooling liquid pressure was approximately 1 bar, and in the experiments presented in this paper was 2 bars.

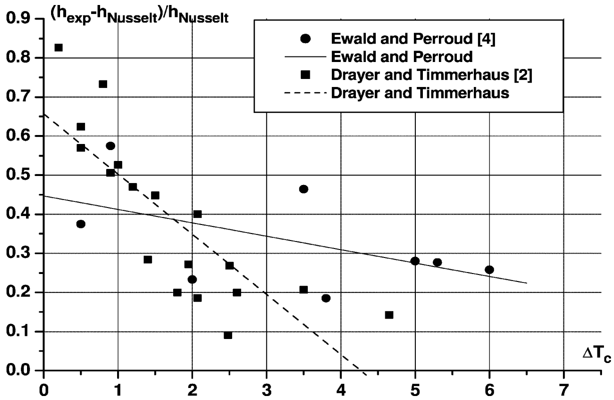

Figure 10. Heat transfer coefficient deviation form Nusselt's theory as a function of the condensate film temperature drop; other works.

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